A Clarification of the Labeling of "Fisher-Pry" Transform Figures

This note refers to the paper **A Primer on Logistic Growth and Substitution: The Mathematics of the Loglet Lab Software**, available as a PDF file on our website: `http://phe.rockefeller.edu/LogletLab/logletlab.pdf`. This paper was also published in the journal *Technological Forecasting and Social Change* 61(3):247-271, 1999. It has come to our attention that we should give a better explanation of how we label the figure axes on some of the figures in the paper. In the paper, we define a "Fisher-Pry" Transform of the standard three-parameter s-shaped logistic curve. We define our logistic slightly different than standard, with the three parameters Δt, which we call the "characteristic duration", κ, the "saturation", and t_m, the midpoint of the logistic curve. The equations from the paper are repeated below for clarity.

$$N(t) = \frac{\kappa}{1 + \exp\left[-\frac{\ln(81)}{\Delta t}(t - t_m)\right]} \tag{5}$$

$$FP(t) = \left(\frac{F(t)}{1 - F(t)}\right), \text{ where } F(t) = \frac{N(t)}{\kappa} \tag{6}$$

$$\ln(FP(t)) = \frac{\ln(81)}{\Delta t}\,(t - t_m) \tag{7}$$

The labeling issue arises on the labels of the right hand side of a semilog plot of the Fisher-Pry transformed logistic. Figure 1 on the following page should make this clear. The top panel shows a logistic curve with $\Delta t = 1.0$, $\kappa = 1.0$, and a midpoint $t_m = 0.0$. t is plotted on the x-axis, $N(t)$ on y-axis. The middle panel shows a figure where the logistic is plotted using the Fisher-Pry transform (equation (6)). t is plotted as above, but the $FP(t)$ is plotted on a $\log_{10}$ based y-axis. The right hand labels are 1% (corresponding to 10^{-2}), 10% (corresponding to 10^{-1}), etc.

Strictly speaking, these labels are accurate to only one digit. To see this, we can combine equations (5) and (6) to get

$$N(t) = \frac{\kappa}{1 + \exp\left[-\ln(FP(t))\right]} \tag{A}$$

On a semilog ($\log_{10}$ based) plot with markings every order of magnitude from 10^{-2} to 10^{2}, equation (A) tells us the actual saturation percentages of κ. For example, when $FP(t) = 10^{-2}$, the saturation percentage of κ is equal to $100 * \frac{1}{1+e^{-\ln(10^{-2})}}$ which to 10 digit accuracy is 0.99009901. The lowest panel in the figure shows the corresponding right-hand-side percentages labeled to 5 digits of accuracy.

Additionally, in the paper we state that "We observe that the time in which the value is between 10^{-1} and 10^{1} is equal to Δt." In an argument similar to the one above, the actual value of the time between 10^{-1} and 10^{1} is $\frac{\ln(100)}{\ln(81)}\,\Delta t$, which is approximately $1.048 * \Delta t$.

We believe our customary style of labeling the axes suits almost all applications of Loglet Lab. For those needing to offer extremely exact representations, we recommend the labeling in the bottom panel of Figure 1.

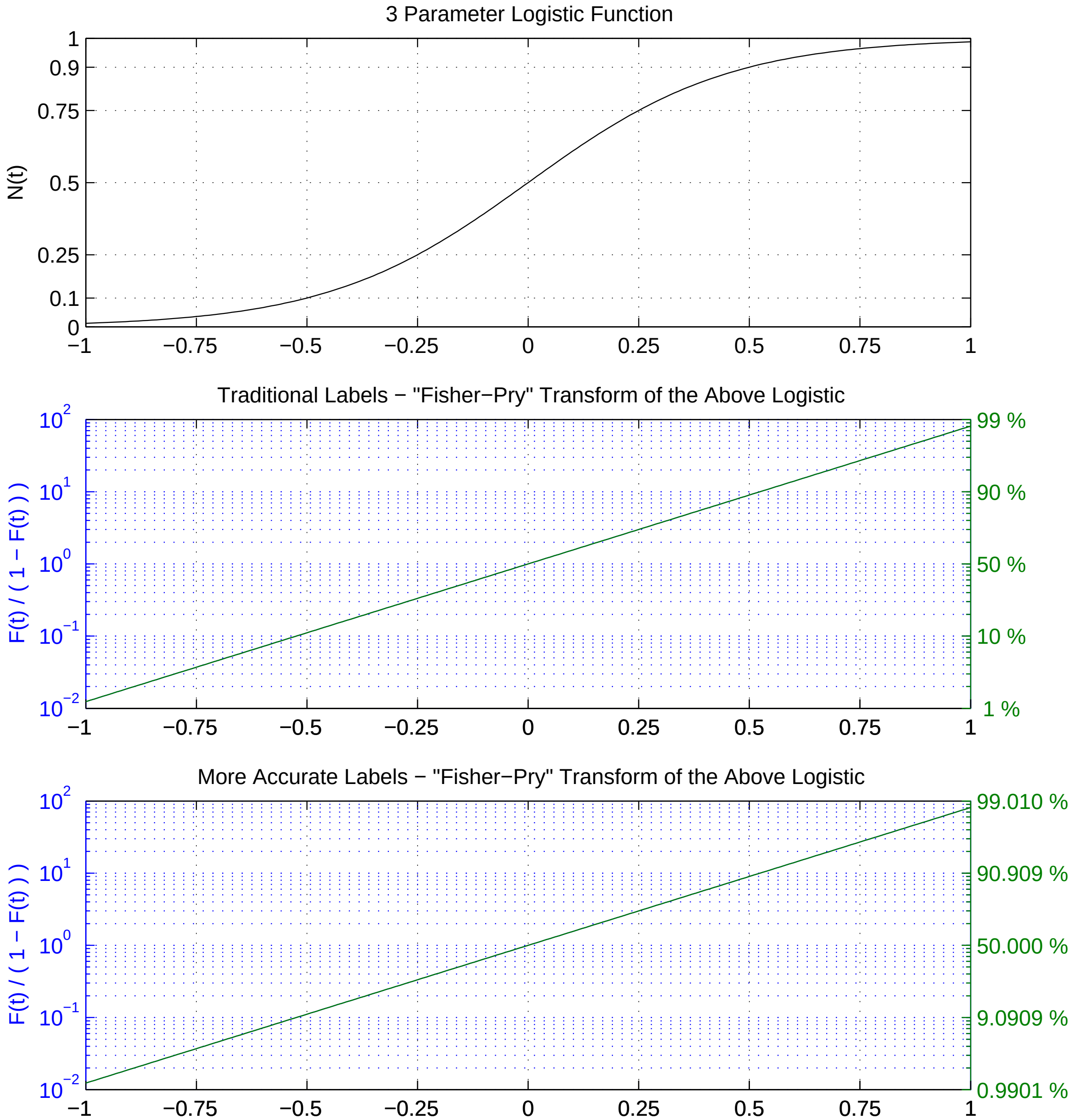

Figure 1. TOP: A three parameter logistic curve; MIDDLE: the Fisher–Pry Transform; BOTTOM: same as MIDDLE but more accurate right–hand labels